



Polygonal Figures and Their Form Spaces

II. Angle Permutations in Equilateral Polygons

A. E. KÖHLER

Lehrstuhl Biophysik, Institut für Biochemie und Biophysik

Friedrich-Schiller-Universität Jena, Philosophenweg 12

D-07743 Jena, Germany

koehler@merlin.biologie.uni-jena.de

(Received December 1997; accepted January 1998)

Abstract—In a closed nonregular equilateral polygonal curve, angles cannot be permuted arbitrarily without opening the curve. Here we investigate the possible angle permutations that leave a polygonal curve closed. To this end we introduce *angle sequences*, i.e., symbol sequences describing the number and arrangement of like angle values in classes of forms. We find that there exist “*universal angle sequences*” giving closed polygons for arbitrary angle values provided that the angle sum is kept constant at one of certain permissible values. The universal angle sequences are *periodic*: they consist of at least two identical series of symbols. Within a series, the symbols can be permuted freely on condition that in all other series the very same permutation is applied. All these *correlated permutations*, when applied to the angles of a corresponding polygonal curve, leave this curve closed.

Polygons belonging to a given angle sequence (form class) constitute a submanifold of the manifold \mathcal{M}_I of equilateral polygons within the form space. These submanifolds structure \mathcal{M}_I in a hierarchical manner so that the form class concept gives a valuable complementary classification scheme for the forms of \mathcal{M}_I .

Furthermore, we find that polygons with constant angle composition (the term “composition” is used here in its chemical sense) show a wide variety of possible shapes (from quasi-circular to wormlike and even self-intersecting ones) corresponding to different degrees of “segregation” of like angles within the polygonal curve. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords—Form spaces, Polygonal forms, Manifolds, Angle permutations.